Théorie de la synchronisation - synchronisation harmonique

Oscillateur soumis à des frottements sec et visqueux

Caractéristiques du système

$T := 0.2 \cdot s$ $\qquad \omega_0 := \frac{2\cdot\pi}{T}$ $\qquad J := 8\cdot 10^{-7}\cdot kg\cdot m^2$ $\qquad q_0 := 270\cdot deg$

Frottement visqueux

$\eta := 0.002$ $\qquad C := 2\cdot J\cdot\eta\cdot\omega_0$ $\qquad F_{v\_max} := C\cdot\omega_0\cdot q_0$ $\qquad F_{v\_max} = 0.015\,N\cdot mm$ $\qquad \lambda := \frac{F_{v\_max}}{J\cdot\omega_0^{\,2}}$ $\qquad \eta_1 := \frac{\eta}{\lambda}$

$\lambda = 0.019$

Frottement constant

$f_b := 0.04$ $\qquad f_1 := \frac{f_b}{\lambda}$ $\qquad F_{c\_max} := \lambda\cdot J\cdot\omega_0^{\,2}\cdot f_1$ $\qquad F_{c\_max} = 0.032\,N\cdot mm$

Calcul du point de synchronisation

$$y_S(\eta, f, a, \varepsilon, y) := racine\left[\left(4\cdot\eta^2 + \varepsilon^2\right)\cdot y^2 + \frac{16}{\pi}\cdot\eta\cdot f\cdot y + \left(\frac{4}{\pi}\cdot f\right)^2 - a^2, y\right] \qquad x_S(a, \varepsilon, y) := arccos\left(\frac{\varepsilon}{a}\cdot y\right)$$

Calcul des paramètres de stabilité

$$s(\eta, f, y) := -\eta - \frac{f}{\pi\cdot y} \qquad p(\eta, f, \varepsilon, y) := \eta^2 + \frac{2}{\pi\cdot y}\cdot f\cdot\eta + \frac{\varepsilon^2}{4}$$

Régimes transitoires vers un point d'arrêt

Excitation harmonique inférieure au seuil d'entretien

$a_{seuil} := \frac{4\cdot f_1}{\pi}$ $\qquad a_{seuil} = 2.702$ $\qquad \boxed{a_1 := 0.8\cdot a_{seuil}}$ $\qquad F_{harm} := a_1\cdot\left(\lambda\cdot J\cdot\omega_0^{\,2}\right)$ $\qquad F_{harm} = 3.217\times 10^{-5}\,N\cdot m$

$n := 500$ $\qquad i := 0..n$ $\qquad x_0 := 0$ $\qquad x_1 := 2\cdot\pi$ $\qquad \Delta x := \frac{x_1 - x_0}{n}$ $\qquad x_i := x_0 + i\cdot\Delta x$

$\varepsilon := -4\cdot\eta_1$ $\qquad \boxed{\varepsilon = -0.424}$

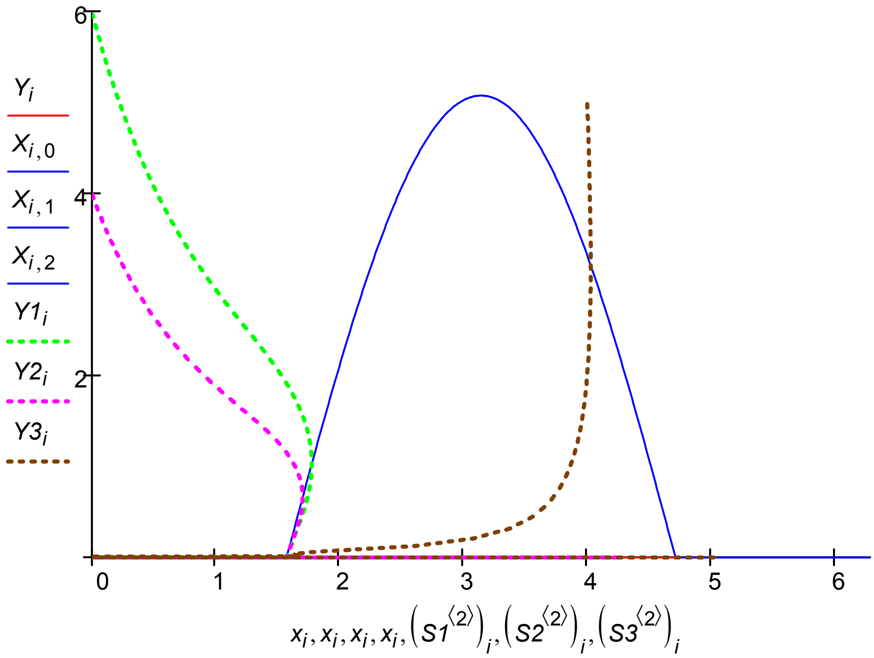
$y := 0$ $\qquad y_s := y_S(\eta_1, f_1, a_1, \varepsilon, y)$ $\qquad y_s = -2.546 + 2.278i$ $\qquad y_s := 0$ $\qquad x_s := x_S(a_1, \varepsilon, y_s)$ $\qquad x_s = 1.571$

$$Y_i := 0 \qquad X_i := \frac{a_1}{\varepsilon}\cdot cos(x_i)\cdot\left(\frac{\pi}{2} < x_i < \frac{3\cdot\pi}{2}\right) \qquad \mathbf{D}(t, Y) := \begin{bmatrix}\left(-\eta_1\cdot Y_0 + \frac{a_1}{2}\cdot sin(Y_1) - \frac{a_{seuil}}{2}\right)\lambda \\ \lambda\cdot\left(\frac{-\varepsilon}{2} + \frac{a_1}{2\cdot Y_0}\cdot cos(Y_1)\right)\end{bmatrix} \qquad t_f := 2000$$

$Y0 := \begin{pmatrix}6\\0\end{pmatrix}$ $\qquad S1 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\qquad Y_1 := S1^{\langle 1\rangle}$ $\qquad Y1_i := Y_{1_i}\cdot\left(Y_{1_i} \geq 0\right)$ $\qquad S1^{\langle 2\rangle} := mod\left(S1^{\langle 2\rangle}, 2\cdot\pi\right)$

$Y0 := \begin{pmatrix}4\\0\end{pmatrix}$ $\qquad S2 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\qquad Y_2 := S2^{\langle 1\rangle}$ $\qquad Y2_i := Y_{2_i}\cdot\left(Y_{2_i} \geq 0\right)$ $\qquad S2^{\langle 2\rangle} := mod\left(S2^{\langle 2\rangle}, 2\cdot\pi\right)$

$Y0 := \begin{pmatrix}5\\4\end{pmatrix}$ $\qquad S3 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\qquad Y_3 := S3^{\langle 1\rangle}$ $\qquad Y3_i := Y_{3_i}\cdot\left(Y_{3_i} \geq 0\right)$ $\qquad S3^{\langle 2\rangle} := mod\left(S3^{\langle 2\rangle}, 2\cdot\pi\right)$



Régimes transitoires vers un foyer attractif pour $\Omega > \omega_0$

Excitation harmonique

$a_{seuil} := \dfrac{4 \cdot f_1}{\pi}$ $\quad a_{seuil} = 2.702$ $\quad \boxed{a_1 := 1.5 \cdot a_{seuil}}$ $\quad F_{harm} := a_1 \cdot \left(\lambda \cdot J \cdot \omega_0^{\,2}\right)$ $\quad F_{harm} = 6.032 \times 10^{-5}\, N \cdot m$

$\varepsilon := -6 \cdot \eta_1$ $\quad \boxed{\varepsilon = -0.637}$

$y := 0$ $\quad y_s := y_S\left(\eta_1, f_1, a_1, \varepsilon, y\right)$ $\quad x_s := x_S\left(a_1, \varepsilon, y_s\right)$ $\quad y_s = 3.405$ $\quad x_s = 2.135$

$s\left(\eta_1, f_1, y_s\right) = -0.304$ $\quad s\left(\eta_1, f_1, y_s\right)^2 = 0.093$ $\quad p\left(\eta_1, f_1, \varepsilon, y_s\right) = 0.155$

$x0_1 := \arcsin\left(\dfrac{4 \cdot f_1}{\pi \cdot a_1}\right)$ $\quad x0_1 = 0.73$ $\quad x0_2 := \pi - x0_1$

$Y_i := \dfrac{1}{2 \cdot \eta_1} \cdot \left(a_1 \cdot \sin\left(x_i\right) - a_{seuil}\right) \cdot \left(x0_1 < x_i < x0_2\right)$ $\quad X_i := \dfrac{a_1}{\varepsilon} \cdot \cos\left(x_i\right) \cdot \left(\dfrac{\pi}{2} < x_i < \dfrac{3 \cdot \pi}{2}\right)$

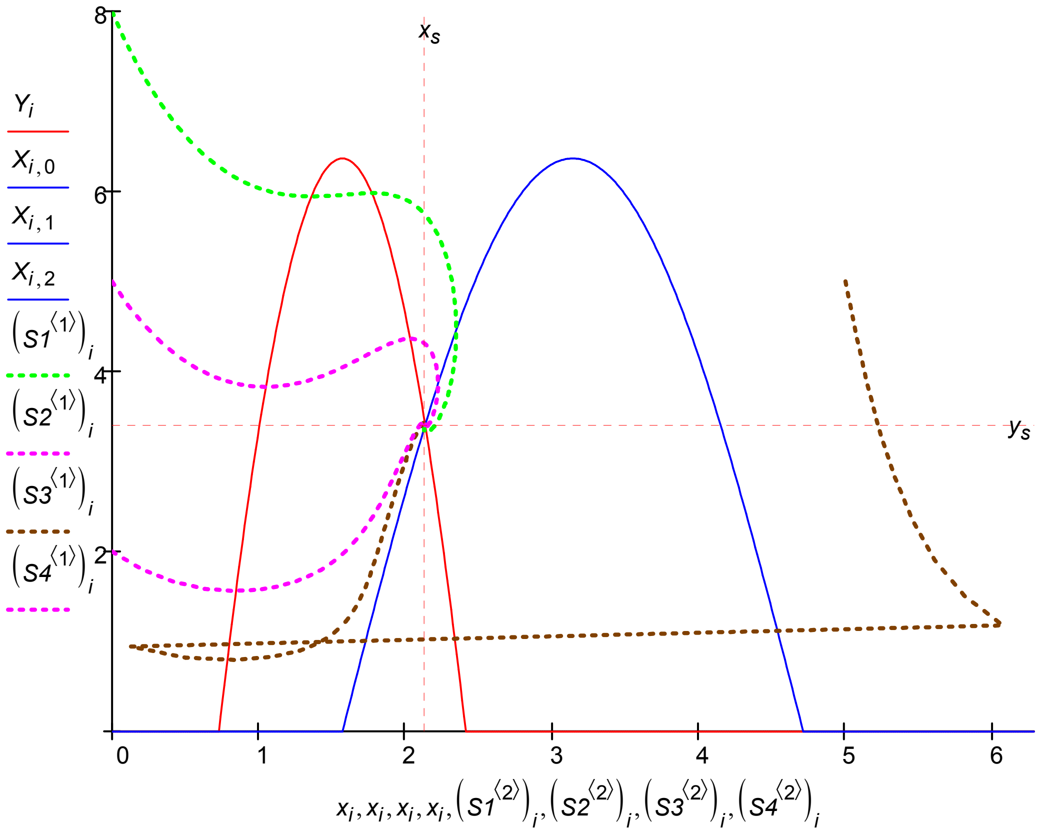
$$\mathbf{D}(t, Y) := \begin{bmatrix} \left(-\eta_1 \cdot Y_0 + \dfrac{a_1}{2} \cdot \sin\left(Y_1\right) - \dfrac{a_{seuil}}{2}\right) \lambda \\ \lambda \cdot \left(\dfrac{-\varepsilon}{2} + \dfrac{a_1}{2 \cdot Y_0} \cdot \cos\left(Y_1\right)\right) \end{bmatrix} \qquad t_f := 4000$$

$Y0 := \begin{pmatrix} 8 \\ 0 \end{pmatrix}$ $\quad S1 := rkfixe\left(Y0, 0, t_f, n, \mathbf{D}\right)$ $\quad S1^{\langle 2\rangle} := mod\left(S1^{\langle 2\rangle}, 2 \cdot \pi\right)$

$Y0 := \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ $\quad S2 := rkfixe\left(Y0, 0, t_f, n, \mathbf{D}\right)$ $\quad S2^{\langle 2\rangle} := mod\left(S2^{\langle 2\rangle}, 2 \cdot \pi\right)$

$Y0 := \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ $\quad S3 := rkfixe\left(Y0, 0, t_f, n, \mathbf{D}\right)$ $\quad S3^{\langle 2\rangle} := mod\left(S3^{\langle 2\rangle}, 2 \cdot \pi\right)$

$Y0 := \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $\quad S4 := rkfixe\left(Y0, 0, t_f, n, \mathbf{D}\right)$ $\quad S4^{\langle 2\rangle} := mod\left(S4^{\langle 2\rangle}, 2 \cdot \pi\right)$



Régimes transitoires vers un noeud attractif pour $\Omega > \omega_0$

Excitation harmonique

$a_{seuil} := \dfrac{4 \cdot f_1}{\pi}$ $\quad a_{seuil} = 2.702$ $\quad \boxed{a_1 := 1.2 \cdot a_{seuil}}$ $\quad F_{harm} := a_1 \cdot \left(\lambda \cdot J \cdot \omega_0^{\,2}\right)$ $\quad F_{harm} = 4.825 \times 10^{-5}\, N \cdot m$

$\varepsilon := -1.5 \cdot \eta_1$ $\quad \boxed{\varepsilon = -0.159}$

$y := 0$ $\quad y_s := y_S(\eta_1, f_1, a_1, \varepsilon, y)$ $\quad x_s := x_S(a_1, \varepsilon, y_s)$ $\quad y_s = 2.437$ $\quad x_s = 1.691$

$s(\eta_1, f_1, y_s) = -0.383$ $\quad s(\eta_1, f_1, y_s)^2 = 0.147$ $\quad p(\eta_1, f_1, \varepsilon, y_s) = 0.076$

$x0_1 := \arcsin\left(\dfrac{4 \cdot f_1}{\pi \cdot a_1}\right)$ $\quad x0_1 = 0.985$ $\quad x0_2 := \pi - x0_1$

$Y_i := \dfrac{1}{2 \cdot \eta_1} \cdot \left(a_1 \cdot \sin(x_i) - a_{seuil}\right) \cdot \left(x0_1 < x_i < x0_2\right)$ $\quad X_i := \dfrac{a_1}{\varepsilon} \cdot \cos(x_i) \cdot \left(\dfrac{\pi}{2} < x_i < \dfrac{3 \cdot \pi}{2}\right)$

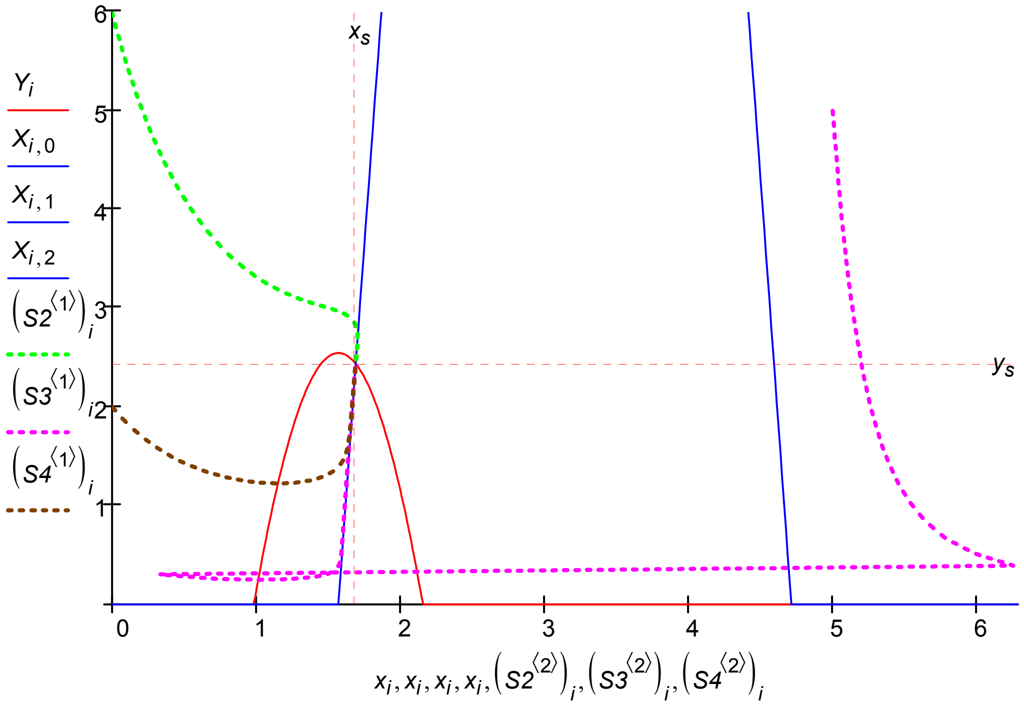
$$\mathbf{D}(t, Y) := \begin{bmatrix} \left(-\eta_1 \cdot Y_0 + \dfrac{a_1}{2} \cdot \sin(Y_1) - \dfrac{a_{seuil}}{2}\right) \lambda \\ \lambda \cdot \left(\dfrac{-\varepsilon}{2} + \dfrac{a_1}{2 \cdot Y_0} \cdot \cos(Y_1)\right) \end{bmatrix}$$

$t_f := 8000$ $\quad k := 2000$ $\quad j := 0 .. k$

$Y0 := \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ $\quad S2 := rkfixe(Y0, 0, t_f, k, \mathbf{D})$ $\quad S2^{\langle 2\rangle} := mod\left(S2^{\langle 2\rangle}, 2 \cdot \pi\right)$

$Y0 := \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ $\quad S3 := rkfixe(Y0, 0, t_f, k, \mathbf{D})$ $\quad S3^{\langle 2\rangle} := mod\left(S3^{\langle 2\rangle}, 2 \cdot \pi\right)$

$Y0 := \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $\quad S4 := rkfixe(Y0, 0, t_f, k, \mathbf{D})$ $\quad S4^{\langle 2 \rangle} := mod\left(S4^{\langle 2 \rangle}, 2 \cdot \pi\right)$



Régimes transitoires vers un foyer attractif pour $\Omega < \omega_0$

Excitation harmonique

$a_{seuil} := \dfrac{4 \cdot f_1}{\pi}$ $\quad a_{seuil} = 2.702$ $\quad a_1 := 1.5 \cdot a_{seuil}$ $\quad F_{harm} := a_1 \cdot \left(\lambda \cdot J \cdot \omega_0^{\,2}\right)$ $\quad F_{harm} = 6.032 \times 10^{-5}\, N \cdot m$

$\varepsilon := 5 \cdot \eta_1$ $\quad \varepsilon = 0.531$

$y := 0$ $\quad y_s := y_S(\eta_1, f_1, a_1, \varepsilon, y)$ $\quad x_s := x_S(a_1, \varepsilon, y_s)$ $\quad y_s = 3.815$ $\quad x_s = 1.048$

$s(\eta_1, f_1, y_s) = -0.283$ $\quad s(\eta_1, f_1, y_s)^2 = 0.08$ $\quad p(\eta_1, f_1, \varepsilon, y_s) = 0.119$

$x0_1 := arcsin\left(\dfrac{4 \cdot f_1}{\pi \cdot a_1}\right)$ $\quad x0_1 = 0.73$ $\quad x0_2 := \pi - x0_1$

$Y_i := \dfrac{1}{2 \cdot \eta_1} \cdot \left(a_1 \cdot sin(x_i) - a_{seuil}\right) \cdot \left(x0_1 < x_i < x0_2\right)$ $\quad X_i := \dfrac{a_1}{\varepsilon} \cdot cos(x_i) \cdot \left[\left(x_i < \dfrac{\pi}{2}\right) + \left(x_i > \dfrac{3 \cdot \pi}{2}\right)\right]$

$$\mathbf{D}(t, Y) := \begin{bmatrix} \left(-\eta_1 \cdot Y_0 + \dfrac{a_1}{2} \cdot sin(Y_1) - \dfrac{a_{seuil}}{2}\right)\lambda \\ \lambda \cdot \left(\dfrac{-\varepsilon}{2} + \dfrac{a_1}{2 \cdot Y_0} \cdot cos(Y_1)\right) \end{bmatrix} \qquad t_f := 4000$$

$Y0 := \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ $\quad S1 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\quad S1^{\langle 2 \rangle} := mod\left(S1^{\langle 2 \rangle}, 2 \cdot \pi\right)$

$Y0 := \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ $\quad S2 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\quad S2^{\langle 2 \rangle} := mod\left(S2^{\langle 2 \rangle}, 2 \cdot \pi\right)$

$Y0 := \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad S3 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\quad S3^{\langle 2 \rangle} := mod\left(S3^{\langle 2 \rangle}, 2\cdot\pi\right)$